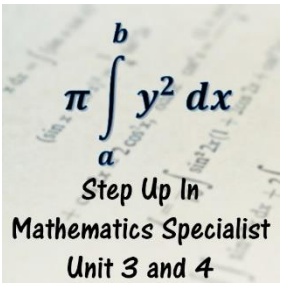
5.3 Differential Equations and Slope Fields

Problems Worksheet



1. Find the general solution to the following differential equations using the technique of separation of variables.

   a. $\frac{dy}{dx} = 4x - 1$

   b. $\frac{dy}{dx} = 3xy$ with $y \geq 0$

   c. $y\frac{dy}{dx} = (x-2)(y^2+1)$

2. Determine the particular solution to the following differential equations using the method of integration by separation of variables.

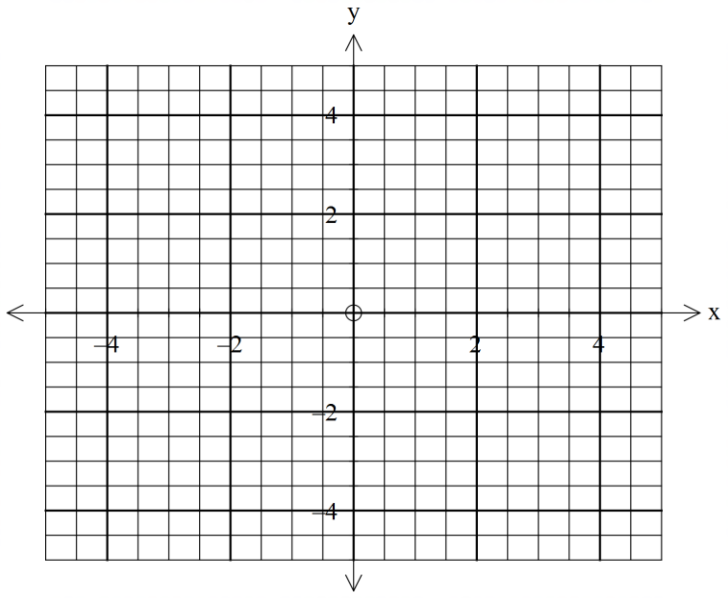
   a. $7x.\frac{dy}{dx} = y^2$ through the point $(e, 14)$

   b. $\frac{dy}{dx} = \frac{5x^3}{1+y}$ through the point $(1, 3)$

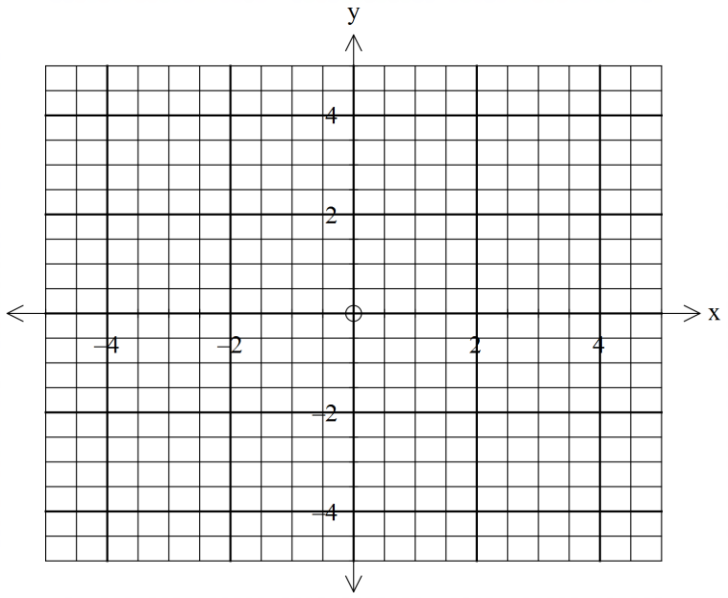
   c. $\frac{dy}{dx} = e^x y^2$ through the point $\left(2, \frac{1}{2}\right)$

3. Sketch the following slope fields and particular solutions.

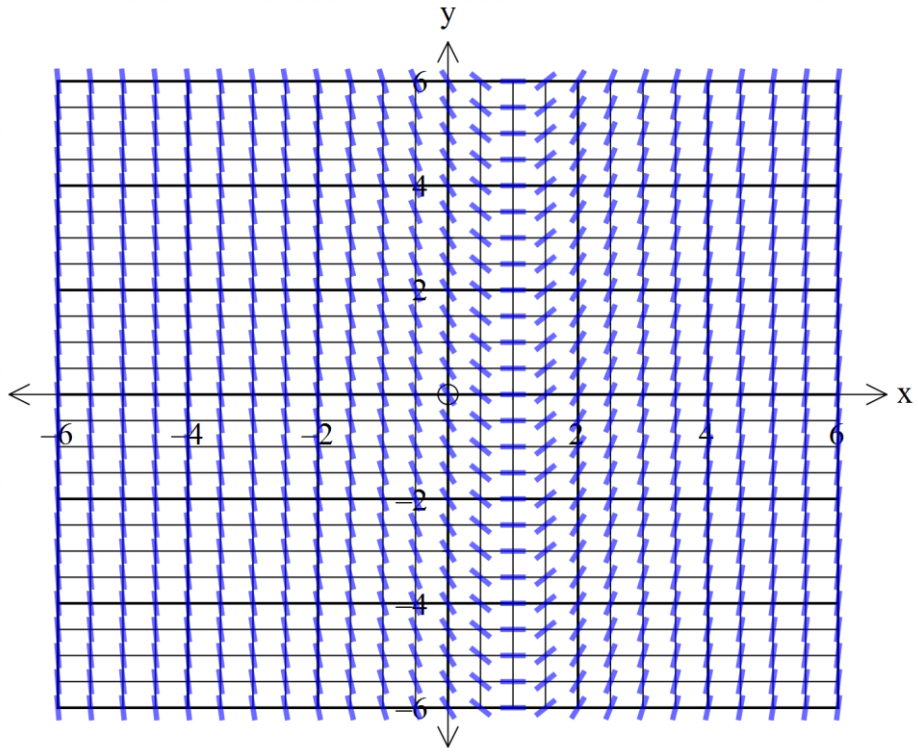
a. $\frac{dy}{dx} = y - x$ with particular solution through $(2, 0)$



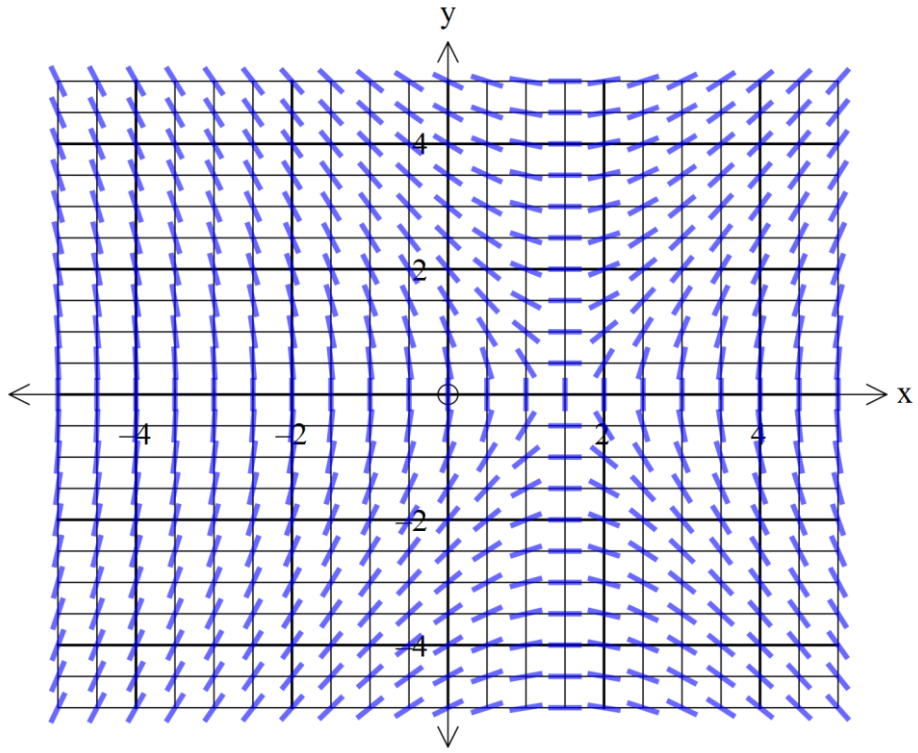
b. $\frac{dy}{dx} = 2x$ with particular solution through $(1, 1)$



4. Below is the slope field for $\frac{dy}{dx} = -a(x-b)$ for some constants a and b. Given that at the coordinate $(2,5)$ the slope is known to be 2, determine the values of a and b and hence find the general solution to the DE and the particular solution through $(2,5)$.



5. Below is the slope field for $\frac{dy}{dx} = \frac{ax+b}{y}$ for some constants a and b. Given that at the coordinate $(-3,1)$ the slope is known to be -9, determine the values of a and b and hence find the general solution to the DE and the particular solution through $(-3,1)$.



6. Use Euler's Method and the suggested step sizes to evaluate the particular solution $y = f(x)$ to each differential equation at the required value of x.

   a. $\frac{dy}{dx} = 2x$ with step size 1 and given $(1, -3)$ on the solution curve, determine $f(4)$.

   b. $\frac{dy}{dx} = 2x$ with step size 0.5 and given $(1, -3)$ on the solution curve, determine $f(4)$.

   c. $\frac{dy}{dx} = \frac{3x}{y}$ with step size 0.5 and given $(4, 4)$ on the solution curve, determine $f(6)$.

   d. $\frac{dy}{dx} = x + y - \frac{y}{2x}$ with step size 1 and given $(-3, -2)$ on the solution curve, determine $f(0)$.

7. Use Euler's Method and the suggested step sizes to evaluate the particular solution $y = f(x)$ to each differential equation at the required value of x.

   a. $\frac{dy}{dx} = \cos x$ with step size $\frac{\pi}{12}$ and given $(0, 3)$ on the solution curve, determine $f\left(\frac{\pi}{2}\right)$.

   b. $\frac{dy}{dx} = \cos x^3$ with step size $\frac{\pi}{12}$ and given $(0, 3)$ on the solution curve, determine $f\left(\frac{\pi}{2}\right)$.

   c. $\frac{dy}{dx} = \cos^2 x^3$ with step size $\frac{\pi}{12}$ and given $(0, 3)$ on the solution curve, determine $f\left(\frac{\pi}{2}\right)$.

   d. $\frac{dy}{dx} = \ln\left(x^2 - \sqrt{x}\right)$ with step size 0.1 and given $(2, e)$ on the solution curve, determine $f(2.5)$.

8. Consider the application of Euler's Method to the differential equation $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$, and the particular solution when it is known that the coordinate $(1, 4)$ lies on the curve. Determine the percentage by which the value of $f(2)$ obtained with the numerical method and a step size of 0.2 exceeds the value of $f(2)$ obtained by integration, where $y = f(x)$ is the solution to the differential equation.